

# Implications of Improved Upper Bounds on $|\Delta L| = 2$ Processes

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## Abstract

We discuss implications of improved upper bounds on the  $|\Delta L| = 2$  processes (i)  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ , from an experiment at BNL, and (ii)  $\mu^- \rightarrow e^+$  conversion, from an experiment at PSI. In particular, we address the issue of constraints on neutrino masses and mixing, and on supersymmetric models with  $R$ -parity violation.

13.20Eb, 14.60Pq, 14.60St, 14.80Ly

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At present there are increasingly strong indications for neutrino oscillations and hence neutrino masses and lepton mixing from the solar neutrino deficiency and atmospheric neutrino anomaly [1]. The existence of lepton mixing means that lepton family number is not a good symmetry. Majorana neutrino masses occur generically, and violate total lepton number  $L$  by  $|\Delta L| = 2$  units. However, so far, in contrast to the data suggesting lepton mixing, experimental searches for the violation of total lepton number have only set limits. Among these are searches for the  $|\Delta L| = 2$  processes (i) neutrinoless double beta ( $0\nu 2\beta$ ) decay of nuclei and (ii)  $\mu^- \rightarrow e^+$  conversion in the field of a nucleus. A third class of  $|\Delta L| = 2$  processes includes the decays  $K^+ \rightarrow \pi^- \ell^+ \ell'^+$ , where  $\ell^+ \ell'^+ = e^+ e^+$ ,  $\mu^+ e^+$ , or  $\mu^+ \mu^+$  [2,3]. In a previous work we considered these decays and, from a retroactive data analysis, set the first upper limit on one of them, namely [4,5]

$$BR(K^+ \rightarrow \pi^- \mu^+ \mu^+) < 1.5 \times 10^{-4} \quad (90\% \text{ CL}). \quad (1)$$

In [4] we also noted that rare  $K$  decay experiments at BNL could greatly improve this limit and proposed a search for  $K^+ \rightarrow \pi^- \mu^+ \mu^+$  [6]. Among these experiments was BNL E865, which was searching for, and has now set a stringent upper limit on, the decay  $K^+ \rightarrow \pi^+ \mu^+ e^-$  [7]. This experiment has also recently obtained the 90 % CL upper limit [8]

$$BR(K^+ \rightarrow \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}. \quad (2)$$

In the present paper we discuss the implications of this limit. In the context of current bounds on neutrinoless double beta decay and  $\mu^- \rightarrow e^+$  conversion, we shall also consider the implications of two other 90 % CL limits on  $|\Delta L| = 2$  decays from E865 [8]:

$$BR(K^+ \rightarrow \pi^- e^+ e^+) < 6.4 \times 10^{-10} \quad (3)$$

and

$$BR(K^+ \rightarrow \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}. \quad (4)$$

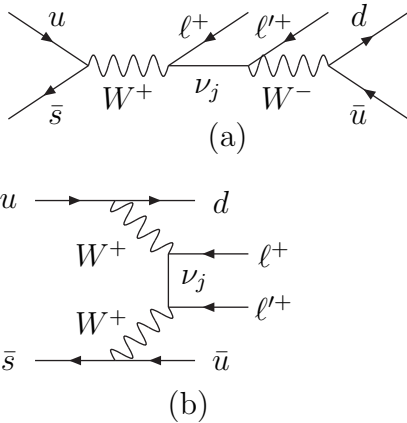
We first discuss physics sources for these decays, concentrating on massive Majorana neutrinos and  $R$ -parity-violating supersymmetric (SUSY) theories. In a modern theoretical context, one generally expects nonzero neutrino mass terms, of both Dirac and Majorana type. Let us denote the left-handed flavor vector of  $SU(2) \times U(1)$  doublet neutrinos as  $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)_L$  and the right-handed vector of electroweak-singlet neutrinos as  $N_R = (N_1, \dots, N_{n_s})_R$ . The Dirac and Majorana neutrino mass terms can then be written compactly as

$$-\mathcal{L}_m = \frac{1}{2}(\bar{\nu}_L \bar{N}_L^c) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + h.c. \quad (5)$$

where  $M_L$  is the  $3 \times 3$  left-handed Majorana mass matrix,  $M_R$  is a  $n_s \times n_s$  right-handed Majorana mass matrix, and  $M_D$  is the 3-row by  $n_s$ -column Dirac mass matrix. In general, all of these are complex, and  $(M_L)^T = M_L$ ,  $(M_R)^T = M_R$ . The diagonalization of the

matrix in eq. (5) then yields  $3 + n_s$  mass eigenstates, which are generically nondegenerate Majorana neutrinos (degeneracies in magnitudes of eigenvalues can yield Dirac neutrinos). Writing the charged current in terms of mass eigenstates as  $J_\lambda = \bar{\ell}_L \gamma_\lambda \nu_L$ , one has, in particular,  $\nu_\mu = \sum_{j=1}^{3+n_s} U_{\mu j} \nu_j$ . The seesaw mechanism naturally yields a set of three light masses for the three known neutrinos, generically of order  $m_\nu \sim m_D^2/M_R$ , and  $n_s$  very large masses generically of order  $m_R$ , for the electroweak-singlet neutrinos, where  $m_D \sim M_{EW}$  and  $m_R \gg M_{EW}$  denote typical elements of the matrices  $M_D$  and  $M_R$ , and  $M_{EW} \simeq 250$  GeV is the electroweak symmetry breaking scale. However, if one tries to fit all current neutrino experiments, including not only the solar and atmospheric, but also the LSND data, then it is necessary to include electroweak-singlet (“sterile”) neutrinos with masses  $\ll M_R$  in order to achieve an acceptable fit. In this case, the weak eigenstate  $\nu_\mu$  may contain significant components of mass eigenstates beyond the usual three light ones  $\nu_j$ ,  $j = 1, 2, 3$ , and, it is *a priori* possible that some of these mass eigenstates might have masses lying in the theoretically “disfavored” intermediate range  $m_D/m_R^2 \ll m_{\nu_j} \ll m_R$  (subject to both particle physics and astrophysical/cosmological constraints). Whether or not such intermediate-mass neutrinos exist is ultimately an empirical question that must be settled by experiment. It is therefore of continuing interest to address constraints from data on neutrino masses in this intermediate mass region. Since current indications from solar and atmospheric data suggest neutrino masses in the seesaw-favored region and since, independent of this, neutrino masses in the range of a few to several hundred MeV can suppress large-scale structure formation and hence may be disfavored by cosmological constraints [9], we concentrate mainly on neutrino masses smaller or larger than this range here.

There are two types of lowest-order graphs involving massive neutrinos that contribute to the decay  $K^+ \rightarrow \pi^- \ell^+ \ell'^+$ , where  $\ell^+ \ell'^+ = \mu^+ \mu^+, \mu^+ e^+, \text{ or } e^+ e^+$ , as shown in Fig. 1.



## FIGURES

FIG. 1. Graphs involving massive Majorana neutrinos that contribute to  $K^+ \rightarrow \pi^- \ell^+ \ell'^+$ , where  $\ell^+, \ell'^+ = \mu^+ \mu^+, \mu^+ e^+, \text{ or } e^+ e^+$ . In the case of identical  $\ell^+$  and  $\ell'^+$ , it is understood that the contributions are from the diagrams minus the same diagrams with the outgoing antilepton lines crossed.

For the  $K^+ \rightarrow \pi^- \mu^+ \mu^+$  decay, the  $s$ -channel diagrams yield, in standard notation,

$$\text{Amp}(2(a)) = 2G_F^2 f_K f_\pi (V_{ud} V_{us})^* \sum_j (U_{\mu j} U_{\mu j})^* p_{K,\alpha} p_{\pi,\beta} \left[ L_j^{\alpha\beta}(p_\mu, p_{\mu'}) - L_j^{\alpha\beta}(p_{\mu'}, p_\mu) \right] \quad (6)$$

where

$$L_j^{\alpha\beta}(p_\mu, p_{\mu'}) = m_{\nu_j} [q^2 - m_{\nu_j}^2]^{-1} \bar{v}(p_\mu) \gamma^\alpha \gamma^\beta P_R v^c(p_{\mu'}) \quad (7)$$

where  $q$  denotes the momentum carried by the virtual  $\nu_j$ , and  $P_R = (1 + \gamma_5)/2$  is a right-handed chiral projection operator. The  $t$ -channel graphs cannot be evaluated so easily, because the hadronic matrix element that occurs,

$$\int d^4x d^4y e^{i(p_d - p_u) \cdot y} e^{i(p_{\bar{s}} - p_{\bar{u}}) \cdot x} \langle \pi^- | [\bar{d}_L(y) \gamma_\beta u_L(y)] [\bar{s}_L(x) \gamma_\alpha u_L(x)] | K^+ \rangle \quad (8)$$

cannot be directly expressed in terms of measured quantities, unlike the matrix elements  $\langle 0 | \bar{s}_L \gamma_\alpha u_L | K^+ \rangle$  and  $\langle \pi^- | \bar{d}_L \gamma_\beta u_L | 0 \rangle$  from the first graph. In the limits where  $m_{\nu_j}^2$  is much smaller or larger than the magnitude of the typical  $q^2$ ,  $|\langle q^2 \rangle_{ave}| \sim O((10^2 \text{ MeV})^2)$ , the propagator factor simplifies:

$$\frac{m_{\nu_j}}{q^2 - m_{\nu_j}^2} \simeq \begin{cases} m_{\nu_j} / \langle q^2 \rangle_{ave} & \text{if } m_{\nu_j} \ll |\langle q^2 \rangle_{ave}| \\ -1/m_{\nu_j} & \text{if } m_{\nu_j} \gg |\langle q^2 \rangle_{ave}| \end{cases} \quad (9)$$

Since the contribution of each virtual  $\nu_j$  is accompanied by a complex factor  $(U_{\mu j}^*)^2$ , it is possible for these contributions to add constructively or destructively. Because of the possibility of such cancellations, one cannot put an upper limit on neutrino masses or lepton mixing matrix coefficients from an upper bound on the decay  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ . (A similar remark applies to  $K^+ \rightarrow \pi^- \mu^+ e^+$  and  $K^+ \rightarrow \pi^- e^+ e^+$  since cancellations can also occur among contributions of various  $\nu_j$ 's to the respective amplitudes for those decays.) The same comment is well-known in the case of neutrinoless double beta decay; for example, in the light-mass region, the lower limit on half-lives for  $0\nu 2\beta$  transitions places an upper limit on  $\sum_j U_{ej}^2 m_{\nu_j}$ , not on  $|U_{ej}|$  or  $m_{\nu_j}$  themselves.

With these inputs, we estimated [4]

$$\text{BR}(K^+ \rightarrow \pi^- \mu^+ \mu^+) \sim 10^{-(13 \pm 2)} r_{\mu\mu} \left| \sum_j U_{\mu j}^2 f(m_{\nu_j} / (100 \text{ MeV})) \right|^2 \quad (10)$$

where

$$f(z) = \begin{cases} z & \text{if } z \ll 1 \\ 1/z & \text{if } z \gg 1 \end{cases} \quad (11)$$

and the factor  $r_{\mu\mu} \simeq 0.2$  is a relative phase space factor (normalized relative to the decay  $K^+ \rightarrow \pi^- e^+ e^+$ ). If  $z \sim O(1)$ , one must, of course, retain the exact propagator. In Fig. 1(a), the range of (timelike)  $q^2$  is  $(m_{\pi^+} + m_\mu)^2 \leq q^2 \leq (m_{K^+} - m_\mu)^2$ , i.e.,  $245 \leq \sqrt{q^2} \leq 388$  MeV. Hence, if there exists a neutrino with  $m_{\nu_j}$  in this range,  $245 \leq m_{\nu_j} \leq 388$  MeV, then  $q^2 - m_{\nu_j}^2$  can vanish, leading to a resonant enhancement of the amplitude for this graph [10].

In the following, we assume for simplicity that a single mass eigenstate  $\nu_j$  dominates the sum in (10) but recall our remark above concerning the possibility of cancellations and note that it is straightforward to generalize our discussion to the case where there are several comparable contributions. If  $m_{\nu_j} \ll m_K$ , then, using the new upper bound (2) and the most conservative choice for the numerical prefactor in eq. (10), (i.e., taking  $10^{-(13 \pm 2)} \rightarrow 10^{-15}$ ), we obtain

$$|U_{\mu j}|^2 \left( \frac{m_{\nu_j}}{100 \text{ MeV}} \right) < 4 \times 10^2 \left[ \frac{BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)}{3 \times 10^{-9}} \right]^{1/2}. \quad (12)$$

On the other hand, if  $m_{\nu_j} \gg m_K$ , we obtain

$$|U_{\mu j}|^2 \left( \frac{100 \text{ MeV}}{m_{\nu_j}} \right) < 4 \times 10^2 \left[ \frac{BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)}{3 \times 10^{-9}} \right]^{1/2}. \quad (13)$$

Since  $m_{\nu_j} \ll m_K$  in order for (12) to hold, and since  $|U_{\mu j}| < 1$  by unitarity, the bound (12) does not place a significant restriction on either of these quantities. Similarly, in the heavy neutrino mass region, since  $m_{\nu_j} \gg m_K$  in order for the bound (13) to hold, it does not place any restriction on  $m_{\nu_j}$  or  $|U_{\mu j}|$  in this mass region [14]. This is not to say, however, that it is not worthwhile to search further for the decay  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ , since it constitutes a testing ground for violation of total lepton number that is quite different from the usual searches for neutrinoless double beta decay of nuclei.

We also give an update of the limit on the decay  $K^+ \rightarrow \pi^- \mu^+ e^+$ . In [4] we obtained an indirect upper limit on this decay by observing that the leptonic part of the amplitude for this decay is related by crossing to the leptonic part of the amplitude for the conversion process in the field of a nucleus  $(Z, A)$ :  $\mu^- + (Z, A) \rightarrow e^+ + (Z-2, A)$ . At the time of [4], the best upper limit on this conversion process was  $\sigma(\mu^- + Ti \rightarrow e^+ + Ca) / \sigma(\mu^- + Ti \rightarrow \nu_\mu + Sc) < 1.7 \times 10^{-10}$  from a TRIUMF experiment [15], and we inferred that  $BR(K^+ \rightarrow \pi^- \mu^+ e^+) \lesssim \text{few} \times 10^{-9}$ . The current best bound, from a PSI experiment, is [16]

$$\frac{\sigma(\mu^- + Ti \rightarrow e^+ + Ca)}{\sigma(\mu^- + Ti \rightarrow \nu_\mu + Sc)} < 1.7 \times 10^{-12} \quad (90 \% \text{ CL}). \quad (14)$$

Using this new bound, we conservatively infer that

$$BR(K^+ \rightarrow \pi^- \mu^+ e^+) \lesssim \text{few} \times 10^{-11}. \quad (15)$$

As was noted in [4], this is an indirect limit since it requires a theoretical estimate of the hadronic matrix element as input. Our indirect bound (15) is more stringent than the E865 limit (4), but the latter limit is still valuable since it is direct.

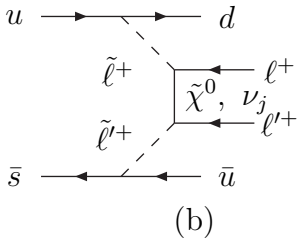
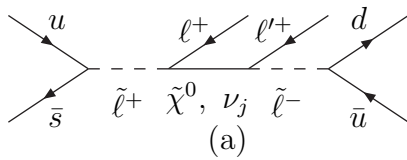
One can obtain an indirect upper limit on the decay  $K^+ \rightarrow \pi^- e^+ e^+$  from the existing upper limit on neutrinoless double beta decay, which, for light neutrinos, gives  $|\sum_j U_{ei}^2 m_{\nu_j}| \lesssim 0.4$  eV (depending on the input used for the nuclear matrix elements) [17]. Using the same method as above, we obtain an upper limit many orders of magnitude less than the direct limit (3).

One can also consider  $\Delta L = 2$  decays of  $D$  and  $B$  mesons. Here only rather modest direct upper limits of order  $10^{-3}$  to  $10^{-4}$  have been set on the branching ratios [12]. A similar comment applies to  $|\Delta L| = 2$  hyperon decays, on which we previously set upper limits [18].

We next proceed to discuss constraints on  $R$ -parity violating ( $RPV$ ) SUSY models.  $R$ -parity may be defined as  $R = (-1)^{3B+L+2S}$ , where  $B$ ,  $L$ , and  $S$  refer to the baryon and lepton numbers and to the spin of the particle [19]. Although  $R$ -parity was originally hypothesized in order to prevent intolerably rapid proton decay in SUSY models, one can achieve the same end by imposing weaker global symmetries that forbid terms of the form  $U_i^c D_j^c D_k^c$  in the superpotential (where the subscripts  $i, j, k$  here are generation indices and we follow the usual convention of writing the holomorphic operator products in terms of left-handed chiral superfields) while still allowing the  $R$ -parity violating terms

$$W_{RPV} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \kappa_i L_i H_u . \quad (16)$$

These terms violate total lepton number and, in general, also lepton family number. A recent review of  $R$ -parity violating SUSY models is [20]. The second term in (16) yields several contributions to  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ , shown in Figs. 2(a,b), where  $\tilde{\mu}$  and  $\tilde{\chi}^0$  denote the scalar muon and neutralino.



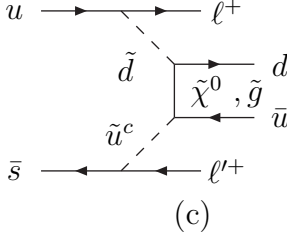


FIG. 2. Graphs that contribute to  $K^+ \rightarrow \pi^- \ell^+ \ell'^+$ , where  $\ell^+, \ell'^+ = \mu^+ \mu^+, \mu^+ e^+, \text{ or } e^+ e^+$ , in supersymmetric theories with  $R_p$  violation. In the case of identical  $\ell^+$  and  $\ell'^+$ , it is understood that the contributions are from the diagrams minus the same diagrams with the outgoing antilepton lines crossed.

Note that there may be neutrino-neutralino mixing in  $RPV$  theories, even if at some mass scale one rotates the terms  $\kappa_i L_i H_u$  to zero. A third type of diagram is shown in Fig. 2(c). As noted, for  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ , each diagram is accompanied by minus the same diagram with the outgoing  $\mu^+$  lines crossed. The  $\tilde{\ell} \tilde{\chi}^0 \mu$  vertices are  $\propto \sqrt{g^2 + g'^2}$ , where  $g$  and  $g'$  are the  $SU(2)$  and  $U(1)_Y$  gauge couplings, while the  $u d \tilde{\mu}$  and  $\bar{s} \bar{u} \tilde{\mu}$  vertices are  $\propto \lambda'_{211}$  and  $\lambda'_{212}$ , respectively and the  $u d \tilde{e}$  and  $\bar{s} \bar{u} \tilde{e}$  vertices are  $\propto \lambda'_{111}$  and  $\lambda'_{211}$ , respectively. In the third type of diagram, there can be a gluino on the internal line, with  $d \tilde{g} d$  and  $\tilde{u}^c \tilde{g} \bar{u}$  vertices proportional to the strong coupling  $g_s$ ; or there can be a neutralino on the internal line, with vertices as given above. Bounds on  $RPV$  couplings are model-dependent, but typical current upper bounds on  $\lambda'_{211}$ ,  $\lambda'_{212}$ ,  $\lambda'_{111}$ , and  $\lambda'_{211}$  are  $\lesssim O(0.1)$  [20]. Using these inputs, we find that these  $R$ -parity violating contributions could be much larger than those from massive neutrinos and lepton mixing, but are still expected to be small compared with the upper limit (2):

$$BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)_{RPV} \lesssim 10^{-16} (\lambda'_{211} \lambda'_{212})^2 \left( \frac{200 \text{ GeV}}{m_{SUSY}} \right)^{10}. \quad (17)$$

For the purpose of this rough estimate, we have taken the masses of the various superpartners  $\tilde{u}$ ,  $\tilde{d}$ ,  $\tilde{\mu}$ ,  $\tilde{\chi}^0$ , and  $\tilde{g}$  to be comparable and denoted this mass scale as  $m_{SUSY} \sim M_{EW}$ . Note that, given the lower bounds on the masses of  $\tilde{e}$  and  $\tilde{\mu}$  or order 100 GeV, no resonance is possible in the amplitude of Fig. 2(a). One could, of course, take a particular SUSY parameter set and perform the calculation for this set; however, there is a large range of variation in possible superpartner masses as well as allowed values of other relevant parameters such as  $\tan \beta$  (as illustrated, e.g., by the parameter sets used in [21]), so we deliberately keep our estimate general. Hence, it appears that the limit (2) does not strongly constrain possible  $RPV$  SUSY theories. The constraints on  $R$ -parity violating models from  $\mu^- \rightarrow e^+$  conversion and neutrinoless double beta decay are also more stringent than those from the limits (4) and (3).

Thus, while neutrinoless double beta decay and  $\mu^- \rightarrow e^+$  conversion are the most sensitive ways to search for  $|\Delta L| = 2$  transitions with  $|\Delta L_e| = 2$  and  $\Delta L_e = \Delta L_\mu = \pm 1$ , respectively, the decay  $K^+ \rightarrow \pi^- \mu^+ \mu^+$  is, at present, the best way to search for  $|\Delta L| = 2$  transitions with  $|\Delta L_\mu| = 2$ . It is therefore worthwhile to estimate the potential of future  $K^+$  decay

experiments to probe for this decay to lower values of branching ratio. In particular this might be undertaken by the CKM experiment planned at Fermilab [22]. The proposal for this experiment anticipates a statistical sensitivity of  $\sim 10^{-12}/\text{event}$  for the decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ . While it would require a substantial, and as yet unplanned, effort to design and build a trigger to search for  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ , it appears [22] that this experiment might be able to reach a level near to  $10^{-12}$  in branching ratio and thus improve substantially on the already impressive upper limit on  $BR(K^+ \rightarrow \pi^- \mu^+ \mu^+)$  from E865 at Brookhaven. We also suggest that the planned experiment MECO at BNL [23], which anticipates searching for  $\mu^- \rightarrow e^-$  conversion below the level  $\sim 10^{-16}$  relative to  $\mu^-$  capture, should also undertake a search for  $\mu^- \rightarrow e^+$  conversion.

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